Test questions on **Limits and Continuity**

**No calculator allowed.**

1. Sketch the graph of a function *f* with the properties $\lim_{x\to 3} f(x) = 4$ and $f(3) = 2$.

2. Sketch the graph of a function *g* with the properties $\lim_{x\to -2} g(x) = 3$ but $g(-2)$ does not exist.

3. Find $\lim_{x\to\infty} \frac{x^2}{e^x}$

4. Describe the graph of the function $\frac{x^2}{e^x}$ as $x \to \infty$?

5. Find $\lim_{x\to -\infty} \frac{x^2}{e^x}$.

6. Consider the function $f(x) = \frac{x-3a}{x^2-5ax+6a^2}$.

   a. Find $f(3a)$

   b. $\lim_{x\to 3a} f(x)$

7. Write an equation of a function with a vertical asymptote at $x = 3$.

8. Write an equation of the asymptote(s) of $f(x) = \frac{3x-7}{x+2}$.

9. What is the range of the function $g(x) = \frac{8+e^x}{4+2e^x}$? Justify your answer.

10. If the minimum value of the function $f(x) = xe^x$ is $-e^{-1}$, what is the range of $f(x)$? Explain your reasoning.

11. Describe the similarities and differences of the graphs of $f(x) = \frac{2}{x-3}$ and $g(x) = \frac{2}{(x-3)^2}$.

12. Describe the continuity of the function $y =$ (the slope of |x|). Justify your answer.

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Answers:

1. **[2 points]** The graph should have a removable discontinuity at the point (3, 4) and an isolated point at (3,2).
2. **[2]** The graph should have a removable discontinuity at (–2, 3).
3. **[1]** 0
4. **[2]** The graph approaches the *x*-axis (the line $y = 0$) as an asymptote.
5. **[1]** The limit does not exist, or the limit is infinity.
6. a. **[1]** does not exist; **[2]** $\frac{1}{a}$
7. **[2]** Various answers are possible such as $y = \frac{1}{x-3}$
8. **[2]** VA: $x = -2$; HA $y = 3$
9. This is not an easy question.
   a. **[1]** as $x \to \infty$ function approaches the asymptote $y = ½$ from above and
   b. **[1]** as $x \to -\infty$ the function approaches the asymptote $y = 2$ from below.
   c. **[2] S**ince $\frac{1}{2} < \frac{8+e^x}{4+2e^x} < 2$ (You may want to prove this.)

   Therefore, the range is ( ½ , 2) or ½ < *y* < 2 (accept an open interval only)
10. **[4]** $\lim_{x\to\infty} xe^x = \infty$ and $\lim_{x\to-\infty} xe^x = 0$ so the range is $\left[-e^{-1}, \infty\right)$ or $x \geq -e^{-1}$ (Accept a half-open interval only).
11. For full credit the student should make the following observations:
    a. **[2]** Both graphs have a horizontal asymptote of $y = 0$
    b. **[2]** Both graphs have a vertical asymptote of $x = 3$
    c. **[2]** The graph of *f* has an odd asymptote at $x = 3$ while the graph of *g* has an even asymptote there. This could be expressed in terms of the appropriate one-sided limits or with a sketch of the graphs.
12. The student should make the points

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a. **[1]** The function is continuous everywhere except $x = 0$, and

b. **[3]** $y = \left(\text{the slope of } |x|\right) = \begin{cases} -1 & x < 0 \\ 1 & x > 0 \end{cases}$ (Could be described in words or with a graph).

A note on the point value: 33 points total.

The AP exams generally award 1 point for each answer or idea tested. I have tried to follow that idea here. So for example in question 2: if a student draws a continuous function through the point (–2, 3) he earns only one point (for interpreting the limit correctly) but does not earn the second point as he did not show a "hole" at that point. Question 9 is a bit tricky (students have to consider limits as $x$ approaches both infinity and negative infinity) so that got an extra 2 points. In question 11 there are 6 observations the student should make.

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